Transonic Drag Effect on Vibration Characteristics of Single-Stage Space Vehicles

Ashok Joshi*
Indian Institute of Technology, Bombay 400067, India

Introduction

HIGH-PERFORMANCE space vehicles experience large drag while passing through the atmosphere, and this drag becomes even larger during the transonic regime leading to larger thrust requirements. Vibration analysis procedures for space vehicles usually take into account the effect of propellant mass depletion and predict structural vibration frequencies that are used for designing the flight controls and other systems. However, the presence of a large drag near the nose of the vehicle and an even larger thrust at the tail leads to the development of significant compressive forces that can have a significant effect on the transverse vibration characteristics. Therefore there is a need to take into account the effect of these compressive forces on structural vibration analysis of space vehicles. It is known that compressive forces reduce the effective transverse stiffness and thereby reduce the natural frequency.[1] Therefore, it can be seen that in the presence of compressive forces, along with mass depletion, the transverse frequencies will be significantly lower than the values predicted by considering only the mass depletion effect. The present study investigates the combined effects of compressive forces and the mass depletion on the transverse vibration frequencies of single-stage space vehicles (used henceforth to denote both missiles and rockets). It is assumed that only the wave drag is significant and is concentrated near the nose portion. The space vehicle is structurally modeled as a nonuniform slender beam with both stiffness and mass properties varying along its length. However, the mass depletion is assumed to be uniform along the length of the vehicle, which is nearly the case for single-stage operations. Further, the trajectory with constant thrust is taken and the axial compressive force P is obtained as a sum of the constant drag force term and a variable inertia force term that is derived from the instantaneous acceleration of the vehicle.

Formulation and Solution

Figures 1a and 1b show the configuration of a typical variable geometry space vehicle. The equations of transverse vibration for a slender beam can be based on the elementary beam theory. In the present case, the governing equation has variable coefficients, and only an approximate solution is possible. Therefore, beam functions in finite constant property segments are used for faster convergence of the solution. This simplification leads to a number of governing equations with constant coefficients that can be solved exactly in terms of the beam functions. The nondimensional equation of motion for ith constant beam segment can then be written as

$$\left(\frac{\partial^4 w_i}{\partial \bar{x}_i^4}\right) - a_i\left(\frac{\partial^2 w_i}{\partial \bar{x}_i^2}\right) + \lambda_i^4 w_i = 0 \qquad (1)$$

Here $a_i\ \{=P(x_i)L_0^2/(EI)_i\}$ is the dimensionless axial compressive force and $\lambda_i\ \{=(\rho A)_i\omega^2 L_0^4/(EI)_i\}^{(1/4)}$ is the dimensionless frequency parameter for the ith segment. Here $\bar{x}_i$ takes values from 0 to $\bar{l}_i$ ($=l_i/L_0$) for all segments where l_i is length of each segment. A new frequency parameter variable λ is defined as

$$\lambda^4 = \left[(\rho A)_0\omega^2 L_0^4/(EI)_0\right] \qquad (2)$$

where ρA_0 ($=M_0/L_0$) is the average mass per length and EI_0 is the maximum bending rigidity. The compressive force $P(x_i)$ for the ith segment is calculated at its c.g. by summing up the drag D inertia force of all of the segments preceding the current one and taking the average inertia force for the current segment, described by the following expression:

$$P(x_i) = (T/M_0)\sum_{1}^{i-1}(\rho A)_j l_j + 0.5(\rho A)_i l_i + D \qquad (3)$$

The general solution of Eq. (1) can be written as

$$w_i = A_i \cosh\lambda_1\bar{x}_i + B_i \sinh\lambda_1\bar{x}_i + C_i\cos\lambda_2\bar{x}_i + D_i\sin\lambda_2\bar{x}_i \quad (4)$$

where A_i, B_i, C_i, and D_i are arbitrary constants and λ_1 and λ_2 (roots of the characteristic equation for each segment) are obtained as

$$\lambda_1^2 = \left[\left(a_i^2 + 4\lambda_i^4\right)^{\frac{1}{2}} - a_i\right]\Big/2 \qquad (5)$$

$$\lambda_2^2 = \left[\left(a_i^2 + 4\lambda_i^4\right)^{\frac{1}{2}} + a_i\right]\Big/2 \qquad (6)$$

The transcendental characteristic equation is obtained by applying the boundary and continuity conditions and is represented by a $4N \times 4N$ determinant (N is the number of segments) whose zeros give various values of the frequency parameter λ. Table 1 presents a convergence study to determine the number of segments required to correctly represent the variable coefficient governing equation. For this purpose, the vehicle is successively approximated with a larger number of spanwise segments, and it is seen that convergence is achieved with eight segments.

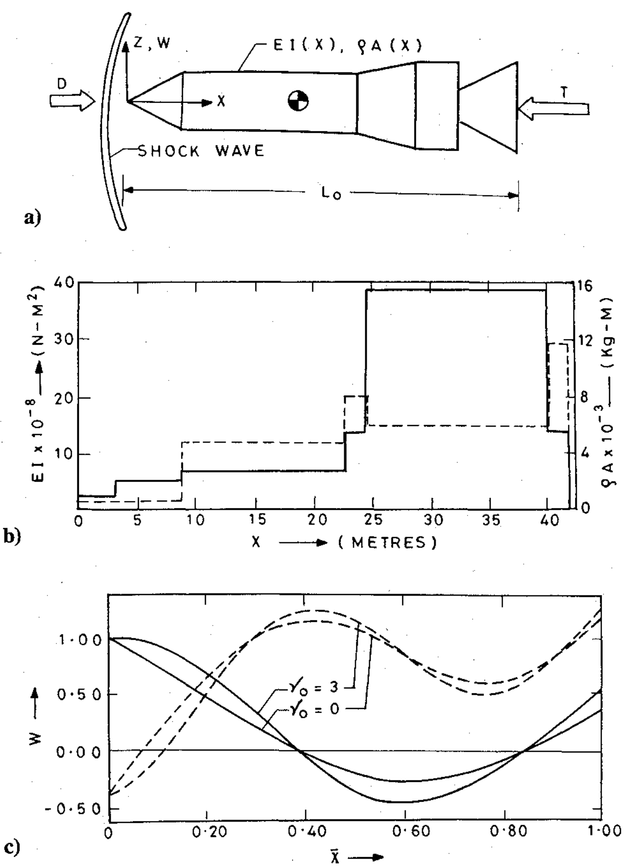

Fig. 1 a) Configuration and geometry of a single-stage space vehicle subjected to thrust and drag; b) ——, the bending rigidity $EI(x)$ and – – –, the mass distribution of a typical single-stage space vehicle; c) ——, first mode and – – –, second mode vibration mode shapes of the typical space vehicle having thrust factor $\beta_0 = 6$, for two values of drag factor $\gamma_0 = 0$ and 6.

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*Associate Professor, Department of Aerospace Engineering, Powai.